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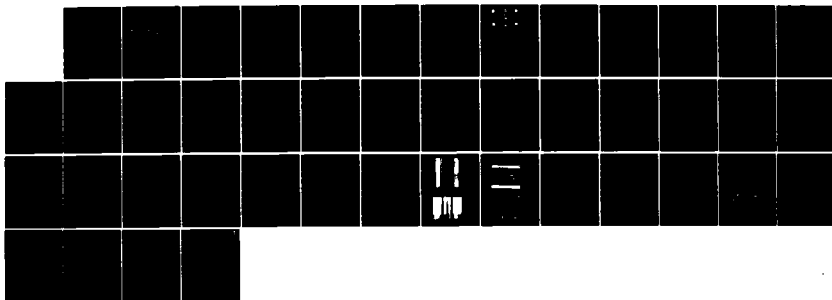
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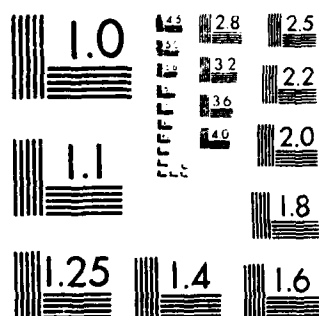
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Report No. 5

Annual Report for Period Aug. 15, 1981 to Aug. 14, 1982

White Light Optical Processing and Holography

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1. Research Objectives

Our research objectives, as stated in the work statement of our proposal, were:

1. Use our white light Fourier transformation process to both construct and use a spatial matched filter, as well as to construct and use other types of complex spatial filters.
2. Analyze the present system limitations and devise ways to improve the system S/N ratio and increase the space-bandwidth product that can be accommodated.
3. Further improve the achromatization through the use of additional dispersive lenses, either refractive or diffractive.
4. Explore methods to extend the achromatization methods to speckle interferometry.
5. Explore various ideas for extending the achromatization methods to spatially incoherent light, that is, to make spatially incoherent light act like coherent light, just as we have made point source white light behave coherently.

2. Accomplishments

We have given attention to all of the objectives given on the previous page. However, our greatest attention was on No. 5, that is, to making spatially incoherent light act like coherent light. This was the most challenging objective, and in our opinion, of the greatest potential value.

Our principal accomplishments for the year have been:

1. We adapted our grating interferometer method for phase-amplitude recording to the area of phase conjugation. We invented and demonstrated a system for compensation of aberrations in an imaging system by holographically recording the wave, generating the conjugate, and sending it through the optical system in a path retracing the original wave. The system

aberrations are therefore compensated. The old way of doing this (which incidentally, we invented in 1965) required coherent light, which is rather noisy. The significant development here is that our new methods enables us to do this with light of considerably reduced coherence, thus reducing the noise this work is described in Sec. 3.

2. We addressed the bias buildup and SNR in incoherent optical processing, making an analysis that indicates that, despite the bias buildup problem in incoherent processing, which can lead to greatly reduced contrast at the output, the incoherent system is generally better than the coherent for SNR. Thus, if we have a sensitive, low-noise detector at the output of an incoherent system, we should do better than if the system were coherent, that is, when we subtract off the enormous bias at the output, we get a very weak signal, but the noise should be, proportionately, even lower. This work is discussed in Sec. 4.

3. A technique for recording in spatially incoherent light the Fourier transform of an object was developed, where the object is the source distribution. This Fourier transform can be spatially filtered in incoherent light, and either the filtered image or the autocorrelation function can be recovered. This idea is discussed in Sec. 5.

4. A new method of recording the Fourier transform, complete with amplitude and phase, in light that is completely spatially incoherent, was discovered. This method uses a three-grating interferometer, adjusted for broad source operation. It has several important aspects; for example, it works on the amplitude transmittance of the object distribution and it preserves the phase, even though the light is incoherent. This achievement is discussed further in Sec. 5.

5. An idea for using the grating interferometer for seeing through turbulence was conceived and is currently being analyzed.

6. Another method for image deblurring was developed and tried. This idea utilizes light of reduced coherence, along with other simple techniques, and results in a very simple and



flexible method for producing high quality deblurred images. This method is described in Sec. 6.

3. Holographic Aberration Compensations with Partially Coherent Light

Aberration compensation by conjugation of a wave is well known in both holography and the relatively new technique of phase conjugation. In the context of holography, a distorting medium, such as an aberrated lens,¹ ground glass,² or shower glass,³ is placed between the object and the recording plate. The hologram records the aberrated wave, and in the readout process, the conjugate wave is generated. This wave traverses in reverse the path of the object wave, compensating for aberrations in the optical path, and forms an aberration-free image, even when the optical system is severely aberrated. The same method has more recently been demonstrated in phase conjugation technology.⁴

The process works extremely well. In an early experiment, ground glass was inserted into the object beam path, completely destroying the image.² Yet, a sharp image was obtained from the conjugate wave of the hologram. However, the process has the shortcoming of requiring coherent light, which is quite noisy. The use of incoherent light removes the coherent noise, but unfortunately the conjugation process, as originally given, requires coherent light.

Clearly, the best of both worlds, the low noise of conventional, incoherent imagery and the aberration compensation of the holographic method would be highly desirable, since exchanging a low resolution, low noise image for a high resolution, high noise one is dubious in most circumstances. Here we describe a technique that allows the compensation process to be carried out with light of greatly reduced spatial coherence. The method is based on the use of broad-source interferometry in off-axis holography, the making of an image-plane hologram rather than a Fresnel or Fourier transform hologram of the object, and the fact that the noise typically

predominates in high spatial frequencies, whereas the aberrations are slowly varying.

By forming an image rather than recording a hologram, the coherence requirements, with respect to the object, are eliminated. If the image contains both phase and amplitude, a spatial carrier can be introduced and thereby permit both to be recorded on photographic film, a process that is like holography, indeed is often termed image plane holography, but in fact goes back to *Ives*⁵. In particular, the use of incoherent light interferometers enables the process to be done with extremely good signal to noise ratio (SNR), since not only does the incoherence reduce noise from scatterers outside the image plane, but also, as opposed to most other methods, does not require any optical elements, such as a diffraction grating, to be placed in the object plane⁶.

Here we combine the phase conjugation method with the incoherent spatial carrier imaging method, producing on the recording medium an image on a carrier as well as a hologram of the aberrations. The spatial coherence of the light is reduced to a degree such that the coherent noise, of relatively high spatial frequency, is substantially reduced, but sufficient coherence is retained so that the system aberration, being of low spatial frequency, is duly recorded as a hologram. The lower the spatial frequencies of the aberration, the better this can be done, since the required coherence in holography depends on the spatial frequencies to be recorded.

An optical system for realization of the method is shown in Fig. 3.1. G_1 , G_2 , and G_3 are diffraction gratings, all of spatial frequency f_1 , forming a three-grating interferometer that produces fringes at the output plane P_0 in light that can be both spatially and temporarily incoherent. GG is a moving diffuser for spoiling the spatial coherence of the laser beam. The upper beam is formed by the +1 order of g_1 , the -1 order of G_2 , and the 0 order of G_3 . The lower beam is formed by the 0 order of g_1 and G_2 , and the +1 order of g_3 . The other orders of the gratings are discarded, or even better, are not produced at all, a feat theoretically possible with the proper construction of holographic gratings. For example, a properly designed thick

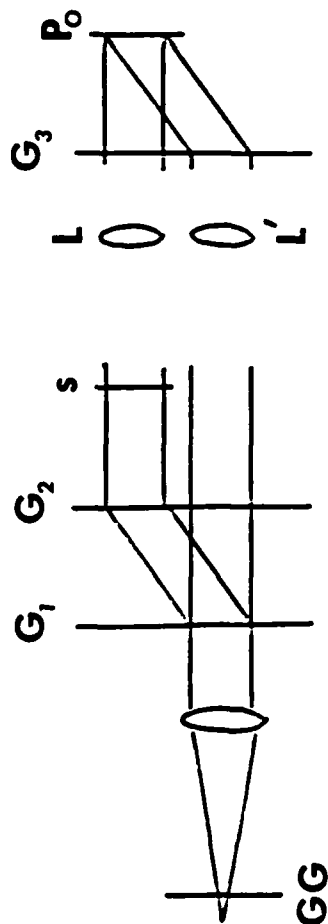


Figure 3.1

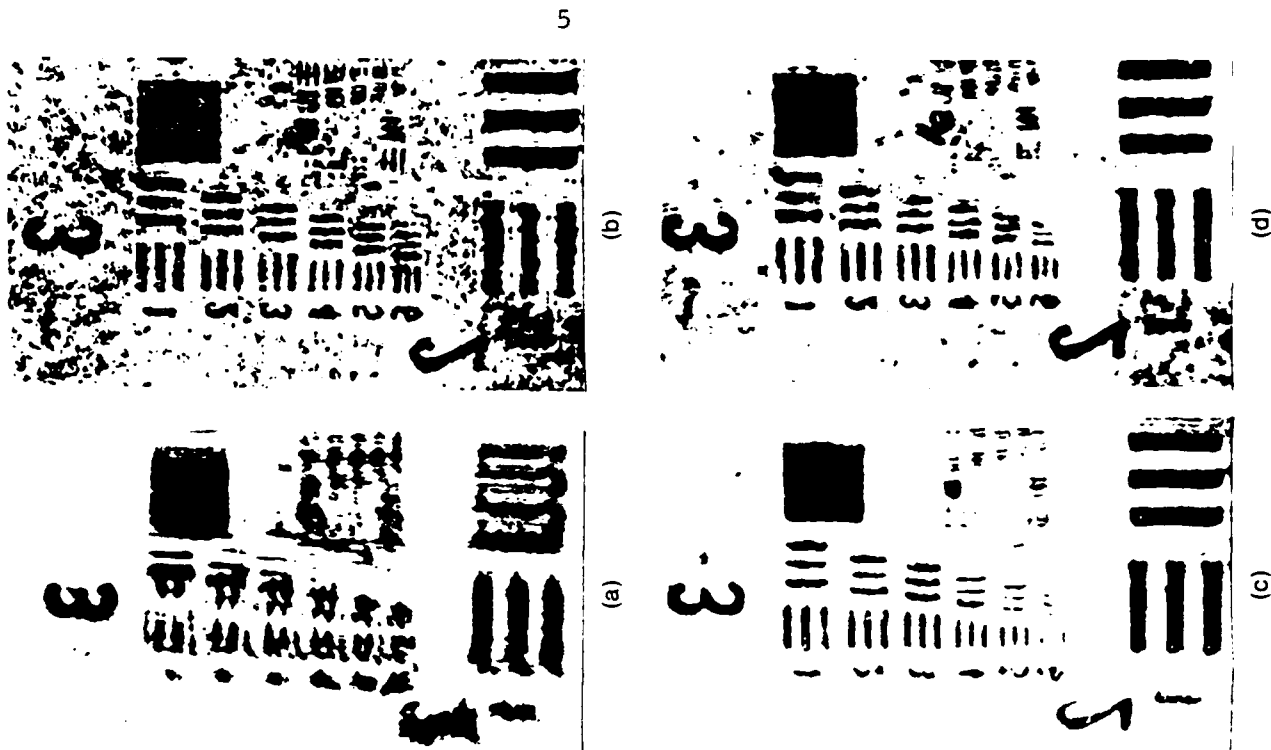


Figure 3.2

holographic grating will exhibit, because of Bragg effects, only a zero order beam and one first order beam. L and L' are a pair of lenses, each inserted into one of the two beams. The upper lens, L, images the object s onto P_0 . The lower beam provides a reference function, so that s is recorded on a carrier. Letting $s = |s| \exp(i\Phi)$, a complex function, the interference pattern at P_0 is

$$|e^{i2\pi f_1 x} + |s| e^{i\Phi}|^2 = 1 + |s|^2 + 2|s| \cos(2\pi f_1 x - \Phi) \quad (1)$$

The phase Φ is accurately introduced into the fringe pattern regardless of the spatial coherence. However, only limited spectral broadening of the source is permitted; otherwise, the phase would be lost, since the phase delay produced by a thickness Δz in the object is $2\pi \Delta z / \lambda$, which is wavelength dependent.

Let the photographic record, in its original position, be illuminated by a duplicate of the reference beam, but traveling in the opposite direction. A beam of light, $|s| \exp(-i\Phi)$, is generated, which then forms an image s^* (or $|s|^2$ in irradiance) at the position of the original object.

Suppose a slowly-varying aberration exists in the object-beam path, produced perhaps by the lens L. For convenience, suppose the phase error to reside at a plane where the Fourier transform of s is formed. This is a common procedure, which simplifies the analysis to that of a spatial filtering operation, with no essential loss of generality. Let the aberration be $U = \exp(i\Phi_a)$. Let the object be illuminated by a monochromatic extended source. Each element of the source produces a beam $\exp(i2\pi f_o x)$ impinging on the object s. For convenience, we drop the third dimension, y. The extension to the third dimension is trivial. The total illumination can then be considered as a summation of incoherent beams, each with a different f_o , related to the angle of incidence by $f_o = \sin(\Theta_o / \lambda) \approx \Theta_o / \lambda$. these various beams are mutually incoherent, and thus sum as irradiances rather than as amplitudes.

Each beam interacts with the object, producing the product $s \exp(i2\pi f_o x)$. At the Fourier transform plane, the field is $S(f_x - f_o)$ where S = Fourier transform of s, and after passing

through the phase aberration, becomes

$$U_o = S(f_x - f_o)U \quad (2)$$

which, at the image plane P_o , becomes

$$u_o = s e^{i2\pi f_o x} * u \quad (3)$$

where u = Fourier transform of U .

The theory of the broad-source interferometer process requires that the function $s * u$ be multiplied by $\exp(i2\pi f_o x)$. If the output image were $(s * u) \exp(i2\pi f_o x)$, it could be regarded as the image of an object distribution $s * u$, illuminated by a plane wave $\exp(i2\pi f_o x)$. The reference beam would similarly contain a term $\exp(i2\pi f_o x)$, and the two beams would then form an interference pattern dependent of f_o , and a broad source could be used.⁸ Thus, we ask, under what conditions can the approximation

$$s e^{i2\pi f_o x} * u \approx (s * u) e^{i2\pi f_o x} \quad (4)$$

be made? Fourier transforming this equation yields

$$S(f_x - f_o)U(f_x) \approx S(f_x - f_o)U(f_x - f_o) \quad (5)$$

or equivalently,

$$U(f_x) \approx U(f_x - f_o) \quad (6)$$

which will be true if U varies sufficiently slowly and if the angular substance of the extended source, and therefore the maximum f_o , is not too great.

A point at x_o on the signal s gives a point spread function

$$u_o = s(x_o)u(x - x_o)e^{i2\pi f_o x} \quad (7)$$

at plane P_o . As before, the exponential term depends on the source element. This is recorded as the hologram component

$$2[s(x_o)|u(x - x_o)|\cos[2\pi f_1 x - \Phi(x_o) - \psi(x_o) - \psi_o(x - x_o)]] \quad (8)$$

where ψ_o is the phase part of u . As before, the interference process has eliminated the source-dependent term $\exp(i2\pi f_o x)$ and the carrier f_1 has come from the reference beam.

The physical description is that each source point projects the signal beam S to a different center position at the Fourier transform plane; thus, the aberration U affects the Fourier transform somewhat differently for each f_0 . For slowly varying U , these shifts can be neglected. The assumption of a slowly varying aberration function is realistic except in the case of rather severe aberrations; consequently, it appears that there may be an optimum source-size region where the source is large enough to give significant noise reduction while still enabling the aberration to be recorded.

The readout process, also carried out with a broad source, is conceptually simpler than the recording process, since a coherent reference beam is not required. One plane wave component of the readout beam is $\exp -i2\pi(f_1+f_0)$; the f_1 term signifies that the beam travels generally in a direction retracing the original reference beam path, and the term f_0 describes the broadening of the source about the average value f_1 . The conjugate beam from the hologram, $s^*(x_0)u^*(x-z_0)\exp -i2\pi f_0 x$, is Fourier transformed to produce

$$s^*(x_0)U(f_x-f_0)e^{i2\pi f_x z_0} \quad (9)$$

which is multiplied by the aberration $U(f_x)$. Again, making the approximation $U(f_x-f_0)=U(f_x)$, we have $UU^* = 1$, the aberration cancels, and an unaberrated image appears at the output.

The concept was verified experimentally in a setup like that shown in Fig. 3.1. The gratings were of spatial frequency $f_1 = 200$ cycles/mm, and the lenses L and L' were identical single-element lenses of focal length 22 cm. The light was obtained by spoiling the spatial coherence of a HeNe laser by the common method of placing a rotating ground glass in the laser beam path. The size of the effective extended source is the size of the laser beam at the plane where it intercepts the diffuser. The lenses were stopped sufficiently that spherical aberration was negligible. Since the other Seidel aberrations are zero for a point object on axis, the reference beam lens L' was essentially a perfect lens. The lens L , since it is imaging a non-zero field,

exhibits other aberrations, particularly astigmatism and coma, which serve as the aberration U. To heighten these, the lens L was skewed, so that the object was off axis.

The size of the source should be a compromise. The larger the source, the more the noise is reduced, but if too large, the aberration U will not be well recorded. Various holograms were made, with various size sources, the size being changed by moving the ground glass axially, thereby changing the size of the beam incident on the glass. The size used in the experiment was $100\text{ }\mu\text{m}$, which subtended an angle of 2.5×10^{-4} radians at the collimator. Since the diffraction-limited image size, seen through the collimator, was 2.4×10^{-5} radians, the redundancy factor due to the broadened source was about 100, the ratio of the actual source size (in area) to the diffraction-limited source size.

Experimental results are shown in Fig. 3.2. First, the reference beam was blocked, and a conventional image, a part of an air force resolution chart, was recorded at P_o with a broad source and a diffuser (Fig. 2a) The aberration is large. Second, with the ground glass removed, the reference beam was introduced, and a hologram was recorded. The hologram was removed, developed, then placed back in its original position, and a readout beam was introduced, identical to the reference beam, but traveling in the opposite direction. The correction achieved is excellent, but the noise is quite high (Fig. 3.2b). Next, the source was broadened to its experimentally-determined optimum and a hologram again was recorded. The reconstruction was made as in Fig. 3.2b, but with a broadened source, of identical angular extent to that used in making the hologram (Fig. 3.2c). The resolution is essentially the same as before, but there is considerable noise reduction; only the low spatial-frequency components of the noise remain. Finally, to see the relative effects of source broadening, in the making and readout steps, the point source hologram was read out with a broad source (Fig. 3.2d). The result, as expected, is a lesser reduction in noise, giving a reconstruction with noise intermediate between the cases of

2b and 2c.

The experimental results can be interpreted as the use of two zero-field, or collimating, lenses being used to give the effect of a conventional, non-zero field imaging lens. Even in this limited context, the technique has considerable potential value, since zero-field lenses can be simple, inexpensive, yet quite good, and imaging lenses of good quality could be many times more expensive. Alternative configurations, in which there is no matching lens in the reference beam may be possible and would give the technique the broader and more general capability of making an inadequate imaging lens into a better one. Also, other interferometers could be used, such as a Mach Zehnder adjusted for broad source fringes. The grating interferometer, however, has two advantages: it is easier to adjust for broad source fringes, and it gives better fringes under broad source illumination.

Also, the method carries over to the related area of phase conjugation, where the potential range of application is far greater because of the *real-time capability of the phase conjugation* process. Initial considerations suggest that alternative experimental setups would be preferred, but that the process should be equally successful in that area.

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4. Analysis of SNR in Incoherent Optical Processing

1. Introduction

Over the past two decades there has been considerable discussion as to the relative merits of coherent and incoherent optical processing (1-3). It has been noted that the incoherent system has many channels for the transmission of the same information, giving a redundancy that leads to noise suppression. Recent analysis by Lowenthal and Chavel supports this conclusion in a detailed and quantitative way (4,5) and suggests that, whenever possible, incoherent processes should be used rather than the coherent.

What kind of optical processing operations can be done incoherently? Essentially, any linear space invariant operation can be done either coherently, or incoherently. Although there are many systems for doing either coherent or incoherent optical processing, there are two that stand out as being rather basic ones, and our discussion is in terms of these. The first (Fig. 1), produces what is sometimes called the spatial domain synthesis, or convolution. The signals is imaged onto a reference function, or impulse response, r , and at the plane P , on axis, the function

$$g = \int s r dxdy \quad (1)$$

is generated. By moving s through the aperture, the output becomes either

a convolution or a cross-correlation, depending on how s is inserted. If a cylindrical lens is used in combination with L_3 , the operation is one dimensional, being either

$$g = \int s(x-x', y) r(x, y) dx \quad (2)$$

or

$$g = \int s(x' - x, y) r(x, y) dx \quad (3)$$

the former being a cross-correlation, and the latter a convolution. For the two dimensional case, a two-dimensional scanning movement of s is required, yielding a 2-dimensional correlation or convolution.

If coherent light is used, the system is linear in amplitude, whereas with incoherent light the system is linear in irradiance. A special case occurs when a point or line source of white light is used; the system then behaves as a coherent system, even though the light is temporarily incoherent.

Complex functions s and r can be handled by placement on a carrier; $s \rightarrow s_b + |s| \cos(2\pi f_o x + \phi)$, $r \rightarrow r_b + |r| \cos(2\pi f_o x + \phi_r)$ where s_b and r_b are bias terms, and ϕ , ϕ_r are the arguments of s and r respectively. This method applies equally whether the system is coherent or incoherent. There are two problems; the output is on a carrier f_o , and is accompanied by a strong bias, $\int s_b r_b dx$. In the coherent case, the bias is removed by spatial filtering at the focal

plane of L_1 and the carrier is removed by a combination of spatial filtering at the focal plane of L_1 and the detection process at the output. For the incoherent case, such filtering is not available; thus the output signal is on a carrier and is accompanied by a bias, both of which have to be removed electronically. The second basic case (Fig. 4.2) is called the frequency domain synthesis or spatial filtering method. A mask at the focal plane of L_1 is a spatial filter, and the output is the input signal modified by the filter. Again, complex values for s and r can be realized by placing s and r on spatial carriers. For the incoherent case, the transfer function becomes the autocorrelation function of the spatial filter mask. It had at one time been believed that, with the incoherent system, arbitrary transfer functions were not possible; the only realizable transfer functions were those that could be written as an autocorrelation function. Lohmann showed that this supposition is not true² and described a basic method for arbitrary complex filters for incoherent spatial filtering. Thus, the incoherent frequency domain synthesis is just as general as the incoherent spatial domain synthesis. The output, as in the spatial domain synthesis, comes with a bias and a carrier, and therefore requires electronic detection to separate the signals from the bias and remove the carrier.

2. Bias Buildup

The bias term is more than just an inconvenience requiring electronic detection and filtering to remove. It can be overwhelming, leading to poor SNR, even though the system incoherence has made the noise level very low. Any comprehensive discussion of the SNR of coherent vs incoherent systems must take into account this bias term.

We discuss the bias buildup in terms of several examples, starting with relatively simple, specific cases and progressing to the more general cases.

In the 1960's, a number of researchers developed methods for making Fresnel or Fourier holograms in spatially incoherent light. Typically, an interferometer was used, which converted every point of the object into a structure

$$a_o^2 [1 + \cos(2\pi f_o + \beta x)] \quad (4)$$

in the case of the Fresnel method, or

$$a_o^2 (1 + \cos 2\pi f_1 x) \quad (5)$$

in the Fourier case. Here, a_o is the object amplitude, f_o is a constant carrier, the same for all object points, β is a constant, and f_1 is a carrier different for each object point. If the object consists of N discrete points, which for simplicity we take to be of equal spacing and equal amplitude, then the Fresnel hologram is formed by recording an irradiance distribution.

$$I = a_o^2 \sum_{n=1}^N \{1 + \cos[2\pi f_o(x - nx_1) + \beta(x - nx_1)^2]\} \quad (6)$$

If we assume all N signals overlap, then the bias term increases or builds up arithmetically, i.e., proportional to N , whereas the root mean square amplitude of the signal builds up as \sqrt{N} , so that the fringe contrast, the ratio of rms amplitude of the fringes to the bias, is proportional to $1/\sqrt{N}$. Thus, high contrast fringes are obtained only for the case of one or a few object points; for the case of several hundred object points, the contrast is vanishingly small. Experience has shown that for more complex objects, such as continuous tone objects, the SNR of the reconstructed image is very poor, despite the expected noise advantage of using incoherent light.

These results can be duplicated using the basic convolving system of Fig. 4.1*. The signal s is an array of slits, representing the delta-function object points. The reference function is a cylindrical off-axis Fresnel zone plate. The output irradiance is recorded as a function of time through a slit at $x = 0$ while the object s is moved through the aperture.

The incoherent convolving system can also be used for the reconstruction process, the compression of the dispersed signals into point images. The hologram previously constructed is

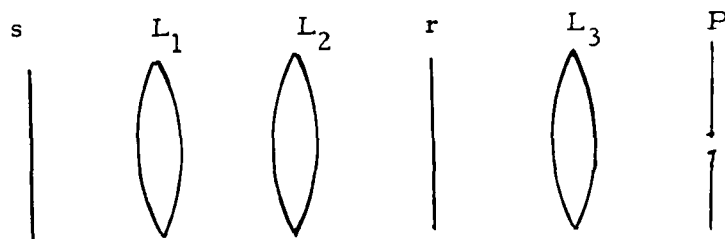


Figure 4.1

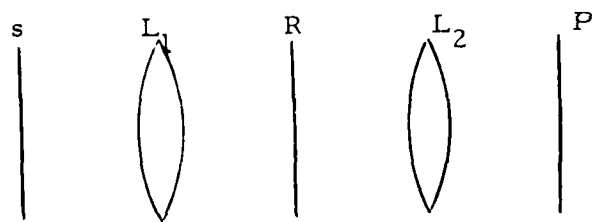


Figure 4.2

used as a signal s , and the same reference function used for making the hologram is used for compression. The bias buildup that occurs in forming the hologram was produced by the overlapping process. Since the present process is one of compressing overlapping signals, one might suspect that the mechanism for further bias buildup is absent, but analysis shows this is not the case.

Let the signal, have transmittance

$$T_s = s_0 + a_0 \sum_{n=1}^N \cos[2\pi f_0(x - nx_1) + \beta(x - nx_1)^2] \quad (7)$$

and the reference function r have transmittance

$$T_r = +\cos(2\pi f_0 x + \beta x^2) \quad (8)$$

where we take $r_0 = 1/2^*$. We place the recorded signal in the system of Fig. 4.1. If we choose coherent operation, then the above functions represent amplitude transmittances. For an incoherent system, the above are to be intensity (or irradiance) transmittances. We form the cross-correlation of the two signals, producing at the output

$$X = s_0 L + \sum L a_0 \operatorname{sinc} \left[\frac{\beta L (x' + x_1)}{\pi} \right] \cos \left[2\pi f_0 (x' + x_1) - \beta x' (x' + x_1)^2 \right] \quad (9)$$

where L is the aperture, or integration interval; this is just the usual holographic image, but on a spatial carrier f_0 and on a bias. The signal to bias ratio is a_0 / s_0 . If we assume that the spacing x_1 is equal to the resolution, this ratio is also approximately the mean signal to bias ratio. The corresponding ratio on the hologram is $\sqrt{N} a_0 / s_0$. Thus, the signal to bias ratio has been reduced in the same manner as in the hologram making process.

The physical basis of this enormous bias buildup can be understood in a rather physical, yet quantitative, way. We consider first the conventional image formation process in a hologram (Fig. 4.3).



Fig. 4.3

Fig. 4.3a shows the conventional hologram reconstruction process. The field on readout consists of a bias term s_b and the image term u , as well as a conjugate term not of interest here. The ratio of mean intensity of the image wave to the bias wave is $|u|^2/s_b^2$. The wave u propagates to the image plane, forming an image s . The energy is unchanged in the process. If we assume for simplicity that the image is the same size as the hologram aperture, the signal to bias ratio is unchanged, since $|s|^2=|u|^2$. By using off-axis holography, the bias term is separated from the image. If the basic Gabor in-line method is used, the bias and image terms overlap.

Next we reconstruct with the convolutional system previously discussed (Fig. 4.3b). Conceptually, we may think of the reference function $+r$ as overlaying the hologram, rather than being imaged on it. The bias term $1/2$ is a multiplier that gives a resultant bias s_b . The r portion is a zone plate, of which the positive focal length term $\exp[-j(2\pi f_0 x + \pi x^2)]$ is used. The portion $\exp[-j(2\pi f_0 x)]$ has a prism effect, bringing the image wave u into the line with the bias term. The quadratic term, in combination with the lens L_3 , constitutes a telescope taking the virtual image, which forms at the front focal plane of the zone plate lens, and reforming it at the back focal focal plane of L_3 , with magnification $M = F_3 / F_{zp}$, where F_{zp} is the focal length of the zone plate lens, and also the focal length of the hologram.

For the one dimensional case (which for convenience is what we consider), the image brightness is reduced in proportion to the magnification, giving a brightness distribution s^2/M , and a mean irradiance s^2/M .

Now consider what the process does to the bias term. As shown in Fig. 4.4 the reference mask attenuates the bias level by a factor $r_0 = 1/2$. The lens L_3 focusses the light into a sinc function of Rayleigh-criterion width $l=2\lambda F_3 0/L$, which we approximate by a rect function of width l .

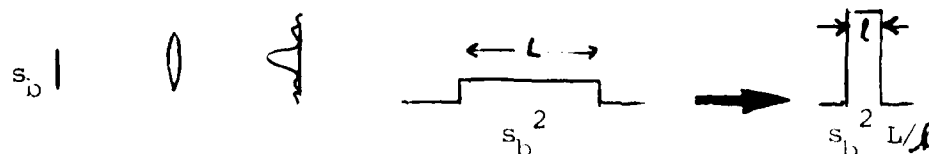


Fig. 4.4.

The increase of brightness is in the ratio $L/l = L^2/2\lambda F_s$, giving a bias irradiance level

$$I_b = s_b^2 L^2 / 2\lambda F_s \quad (10)$$

The signal to bias irradiance ratio is then

$$R = \frac{s^2 F_m / F_s}{s_b^2 L^2 / 2\lambda F_s} \quad (11)$$

The zone plate focal length F_m is $\pi/\lambda\beta$, giving

$$R = (s^2/s_b^2) (2\pi/\beta L^2) \quad (12)$$

The space bandwidth product of an image point, as recorded on the hologram (or equivalently, of r) is readily shown to be $N_s = \pi\beta L^2/\pi$ giving

$$R = (s^2/s_b^2) N_s \quad (13)$$

We thus see a large bias buildup factor, equal to the space-bandwidth product of the point spread function of the process performed.

Physically, what has happened is rather simple. The system has conducted the signal part to the output plane with essentially no change in brightness beyond that produced by a magnification factor. The bias term, however, has been concentrated into a small region, with a consequent buildup in intensity proportional to SW. This bias term remains fixed at the sampling point for the entirety of the processing, while the processed image crosses the sampling aperture, one resolution element at a time.

With coherent illumination, the bias term can be removed by various means, including spatial filtering at the focal plane of L_1 , or a sideways displacement of the sampling slit so as to avoid the bias term. However, when the illumination is incoherent, neither option is available. The usual way to remove the bias is then by temporal filtering of the detector output, an option made possible because when the input signal and reference function signals are on a spatial carrier, the output will be on a temporal carrier. The process requires detecting a low contrast

signal, a process that can be quite noisy. The bias term traveling through the system will generate artifact noise from scatterers in the optical system, and signal-dependent detector noise will be enormously increased. Thus, the lower noise level of the incoherent system is offset by the decreased signal level. If the bias buildup factor dominates, then, the incoherent method is in fact worse than the coherent.

The above results can be obtained on a basis divorced from optics. We consider the convolution process

$$(s_b + s) * (r_b + r) = \int_{-L/2}^{L/2} s_b r_b + F\{(SR)\} \quad (14)$$

where, for $r_b = 1/2$, the first term becomes $L s_b / 2$.

Now, let

$$\begin{aligned} r &= \cos(2\pi f_o x + \beta x^2) \\ &= 1/4 \exp j(2\pi f_o x + \beta x^2) + 1/4 \exp -j(2\pi f_o x + \beta x^2) \end{aligned} \quad (15)$$

The Fourier transform of r is

$$R = 1/4 \frac{\sqrt{\pi}}{\sqrt{\beta}} \left[\exp -j \frac{(f-f_o)^2}{4\beta} + \exp j \frac{(f+f_o)^2}{4\beta} \right] \quad (16)$$

or, for the single sideband case,

$$R = 1/4 \frac{\sqrt{\pi}}{\beta} \exp -j \frac{(f-f_o)^2}{4\beta} \quad (17)$$

After filtering, the signal energy is attenuated by a factor $\pi/16\beta$, or $\pi/8\beta$ for the double sideband case. The signal to bias ratio of the output is then

$$\frac{I_{sig}}{I_{bias}} = \frac{\bar{s}^2 \left(\frac{\pi}{8\beta} \right)}{1/4 \bar{s}_b^2 L^2} = \frac{\bar{s}^2}{\bar{s}_b^2} \frac{\pi}{2\beta L^2} = \frac{\bar{s}^2}{\bar{s}_b^2} \frac{1}{2N_b} \quad (18)$$

which, (except for a factor 2) agrees with the previous result.

This result can be demonstrated on a rather general basis. Let the Fourier transform of r be R . From Parseval's theorem we have

$$\int_{-\frac{L}{2}}^{\frac{L}{2}} r^2 = \int_{-\frac{W}{2}}^{\frac{W}{2}} |R|^2 \text{ or } L \bar{r}^2 = W |R|^2 \quad (19)$$

For a high contrast mask, $r_b + r$, with $r_b = 1/2$, we expect \bar{r}^2 to be of the order of $1/4$, giving

$$|R|^2 = L/4W \quad (20)$$

The filter R thus gives an irradiance attenuation of $L/4W$. If we take the statistics of the output signal s_o to be the same as that of the input signal s , then

$$\bar{s}_o^2 = L \bar{s}^2 / 4W,$$

and we have for the signal to bias irradiance ratio

$$\frac{I_{sig}}{I_{bias}} = \frac{\bar{s}^2 (L/4W)}{1/4 \bar{s}_b^2 L^2} = \frac{\bar{s}^2}{\bar{s}_b^2} \quad (21)$$

The full benefit of the reduced noise resulting from the incoherent illumination is thus fully realized only for processing operations where the space bandwidth product is unity. The most commonplace example is conventional image formation. The bias buildup phenomenon is entirely absent, regardless of the system resolution, and it is clear that the use of coherent illumination would be foolish in all but exceptional circumstances, such as the unavailability of incoherent illumination. Other operations, which involve a modification of the image corresponding to an r with space bandwidth product of the order of one, include image subtraction, pseudocolor generation, placement of a signal onto a spatial carrier (i.e., image plane holography), phase contrast recording, etc. Excellent successes have been reported using incoherent light for these processes.

On the other hand, operations that have a large space bandwidth product, such as spatial matched filtering and image deblurring, would appear to be best done in the coherent mode, using spatial incoherence in one dimension if the operation is one dimensional, or using temporally incoherent light in the achromatic, or coherent processing mode.

A special case arises when the operation to be performed is a Fourier transform, since this operation is not characterized by a point spread function h that is convolved with the object. However, one can proceed in an analogous manner and get similar results. Let the object be of extent L , and let l be either the size of the smallest detail on the object, or the smallest detail that the optical system can resolve. For the latter case, $l = \lambda F/A$, where F is the focal length and A the aperture of the lens. The spatial frequency bandpass of the lens is thus $W = A/\lambda F$, and the space bandwidth product of the system (object aperture plus lens) is $N = LW = LA/F = L/l$. If l is limited by the fineness of object detail, the result is the same, except that the space bandwidth product N is attributed to the object rather than to the object-lens combination.

A number of methods exist for performing a Fourier transform with incoherent light. An object resolution cell centered at x_0 thus transforms into a cosine transform $+\cos 2\pi f_x x_0$, 0 instead of into $\exp j2\pi f_x x_0$, as in the coherent case. Again the bias buildup occurs, with the number of overlapping responses being the space bandwidth product N .

The reduction in processed image contrast must be considered when calculating the SNR. The gain, assuming fully incoherent illumination, is⁵

$$G = \sqrt{N} \quad (22)$$

where N is the space bandwidth product of the optical system.

$$N = S_o S_p / \lambda^2 d^2 \quad (23)$$

where S_o is the area of the object, S_p is the area of the pupil plane, and d is the lens focal length.

The gain, when considering the contrast loss, becomes

$$G' = \sqrt{N}/N_h \quad (24)$$

The largest possible impulse function space bandwidth product, N_h , that can be achieved in the optical system is N . Thus, for this case, there is no gain at all with the use of incoherent light.

On the other hand, if we consider that N for an optical system is generally of the order of 10^6 ,

whereas N_h , for a typical matched filtering operation, is of the order of 10^5 , and for linear deblurring operations, of the order of 10, it is apparent that in principle, gains of the order of $G' = 10$ to 100 are available for these applications. However, these improved SNRs will be accompanied by a greatly reduced contrast, so that, with the decrease of signal (or setup) noise, the detector noise becomes of increased concern.

SNR improvements of the order 10^2 or 10^3 essentially eliminate setup noise as a problem, but the low contrasts result in detection problems. Finer grain and therefore slower films are required, or perhaps very low noise non-photographic detectors are required.

3. Comparison with Spectroscopy

The choice between coherent and incoherent optical processing is thus not at all clear-cut. One might ask the question, are there any optical processing areas where both coherent and incoherent optical processing have been extensively considered. There are perhaps two.

The first is the optical processing of synthetic aperture radar data, in which a complex, holographic-like signal is convolved with a complex reference function that is zone-plate-like in structure. The earliest-conceived processors, of the 1954 period were incoherent. In the following years, coherent optical processors dominated, but many incoherent-type optical processors were conceived^{10,11}. For the most part, these were never highly developed, and even today, with the emphasis on incoherent processing, are in no way a challenge to the coherent. The principal reason is that the noise improvement through redundancy is introduced in another way. The systems collects more data from each object point than can be coherently processed, so the actual processing, although fundamentally coherent, is in fact a mixture of coherent and incoherent, integrated together in a process called tracking¹². It is remarkable that the resulting radar imagery, although produced from data that was obtained by a coherent process (a

coherent radar) and processed in a coherent optical system, using a point source of laser light, is completely free from the artifact noise associated with coherent systems.

Another, far more interesting example of the competitiveness of two modes is found in spectroscopy. This is perhaps the most important example of optical processing, and it exists in two basic and important forms,--grating spectroscopy and interferometric spectroscopy. They have each found a niche and are highly complementary. We would like to identify the former with coherent processing and the latter with incoherent.

In interferometric spectroscopy, an interferometer, such as a Michelson or a variant thereof, splits a beam from an extended source into two parts and recombines them with a time displacement. Each wavelength component λ_0 is converted into a response

$$I(\tau) = I_0 \left[1 + \cos 2\pi c \frac{\tau}{\lambda_0} \right] \quad (25)$$

or

$$I(x) = I_0 \left[1 + \cos 2\pi \frac{x}{\lambda_0} \right] \quad (26)$$

where τ is the time difference between the two beams, and $x = c \tau$ is their displacement. For a multiplicity of wavelengths, the output is a summation of terms of the form of Eqs. (25) or (26), and for a continuous distribution, the irradiance is

$$I_\tau = \int_0^\infty L(\lambda) d\lambda + \int_0^\infty L(\lambda) \cos 2\pi \frac{x}{\lambda} d\lambda \quad (27)$$

where L is the irradiance spectral density distribution. The wavelength resolving process is completed by detecting the signal thus generated, with subsequent Fourier transformation in a digital computer. This process is obviously an incoherent operation, and is in fact wholly analogous to the previously noted forms of incoherent holography, whereby an incoherent object distribution is split into two images by an interferometer and each point in the one image produces a beam that interferes with the beam from the corresponding point of the other image.

The grating case is more subtle. The formation of a wavelength spectrum with a grating and lens, or a curved grating, with its own focal power, an obvious analog to a Fourier transform of a spatial signal with a coherent light beam, is described by the operation

$$I_t = \int_0^\infty L(\lambda) d\lambda + \int_0^\infty L(\lambda) \cos \frac{2\pi}{\lambda} x d\lambda \quad (27)$$

where $L(\lambda)$ is the irradiance spectral distribution of the source, $\exp j2\pi f_0 x$ represents the spatial Fourier component of the grating used for dispersing the light, A is the system aperture, and the final term is the Fourier kernel produced by a lens of focal length F .

If we apply the linearity condition that if $f_i \rightarrow g_i$, then $\sum f_i \rightarrow \sum g_i$, where f_i are the inputs and g_i the outputs, and if we assume that the f_i 's are not overlapping as functions of λ , but that the g_i 's are, then clearly the system is quite linear in irradiance and is thus a wholly incoherent system. On this basis, both types of spectroscopy techniques are incoherent; in terms of our context, they differ in that the grating method is an operation with a unity space bandwidth product at the output recording plane, whereas the interference method has a space bandwidth product much greater than one. Thus, the one is afflicted with bias buildup, whereas the other is not.

However, the grating case has an obvious similarity to the coherent optical Fourier transformation of a spatial signal with a coherent light beam, and the illumination source, being a slit, has spatial coherence. The system is in fact a summation of many such Fourier transformations. In particular, for the case of a discrete spectrum, where the responses g_i have no overlap, the issue of whether the system is coherent or incoherent becomes moot, and the system can be perfectly well regarded as a coherent Fourier transforming device. Indeed the substitution $f \lambda F = y$ and $f_t \lambda F = z$ into Eq. 27 yields

$$\int \left[\int \sqrt{L(y)} e^{2\pi i y} \right] e^{2\pi i z} df_t \quad (28)$$

where $L'(y) = L(y/Ff_0) = L(\lambda)$. The bracketed term is the Fourier decomposition of a function $f(f_t)$:

thus, the integral can be written

$$\int d(f_z) e^{2\pi i f_z z} df_z \quad (29)$$

which is the in the form of the Fourier transform of a spatial function. Thus, although an incoherent system, the grating spectroscopy has a considerable resemblance to a coherent system, and under certain very realistic circumstances can be treated like one.

Of principal interest is how the noise, particularly the setup noise, behaves in such a system. We show that with respect to such noise, the system performance closely parallels that of an achromatic coherent. Optical processing system^{13,14}. In such a system (Fig. 4.5), the

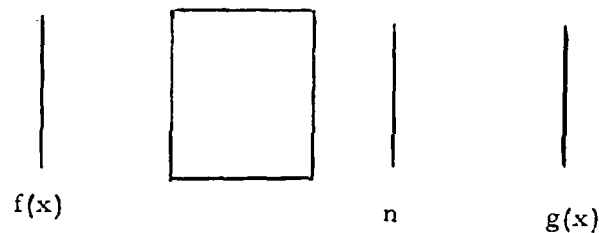


Fig. 4.5

transformation that carries f to g is identical over a range of wavelengths, so that the output g formed from one wavelength is in registry with that formed from the other wavelengths. However, a noise signal n , residing in another plane, produces at the output a wavelength-dependent spatial structure that becomes smoothed when the illumination is spectrally broad. Thus, setup noise is diminished.

The achromatic coherent optical processor can assume many forms, depending on the application. However, the above observation is always true and is the essence of these systems.

The mismatch variation may be in the form of a scale factor change, which produces a spatial-frequency-dependent and a sometimes position-dependent smoothing effect; low spatial frequencies may not be adequately smoothed. It was shown (15) that when the optical system contains a diffraction grating, for example in the form of a signal f residing on a spatial carrier, there will be, in addition to a wavelength scaling effect, a lateral dispersion which can enormously increase the smoothing effect of the polychromatic light. Looking back at the source through the grating, one sees the source broadened into a spectrum; the effectively broader source gives a reduced lateral (i.e., spatial) coherence. Thus, by use of the grating, temporal incoherence is partially converted into spatial incoherence. It is well known that spatial incoherence gives better noise suppression than temporal incoherence. Thus, the grating improves the noise suppression capability of the spectrally broad spatially coherent light.

The reduction in noise is readily seen by the following analysis, which considers the case where the smoothing is by λ -dependent lateral displacement; other cases can be similarly analyzed. The field impinging on the noise plane, for a single wavelength, can be written u . Letting the transmittance of the noise plane be $1 + n$, where $n \ll 1$, the light distribution at the output plane is $u' (1 + n')$, where u' and $u' n'$ are u and un propagated to the output plane. The irradiance distribution is

$$I = |u'|^2 + |u'|^2 (n' + n'^*) \quad (30)$$

If the system has N degrees of freedom, produced by spectral broadening of the source, that is, if N is the number of resolvable wavelengths in the source, as seen through the optical system, then each wavelength component produces a pattern of the form of Eq. 30. The constant term builds by a factor N , the fluctuating term by a factor \sqrt{N} giving a noise reduction factor of \sqrt{N} .

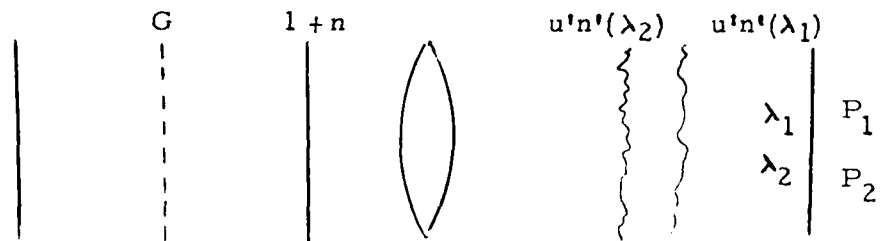


Figure 4.6

In the grating spectrometer case, the light impinging on the noise plane is brought to focus. Again, we take the source to have N resolvable components. Impinging at a point P_1 (Fig. 6) are the direct light for the wavelength λ_1 , as well as the noise light for all N wavelengths. Only the noise light $u' n'(\lambda_1)$ interferes with the direct light to produce a term of the form of Eq. 30. The other noise terms do not, and their noise contribution is hence negligible.

To relate this system to the optical processing case, we compare this system with a hypothetical system in which all of the $u' n'$ interfere with the λ_1 light. This could be realized, for example, by replacing the broad spectrum slit source with a spatially broad coherent source. Such a system, although performing no useful function we can think of, does serve as the fourth element of our comparison. Thus, in terms of noise, the achromatic optical processing system is to the coherent as the grating spectrometer is to this broad-source system. The noise suppression in the two broad spectrum cases is \sqrt{N} compared to the monochromatic counterparts.

The direct identification of two basic kinds of optical processing with the two major kinds of spectroscopy is potentially significant, since a vast body of research in spectroscopy becomes relevant to the smaller area of optical processing. It would have perhaps been more significant if the purely coherent optical processing, as opposed to the achromatic coherent, could have been tied to spectroscopy, but it appears that the purely coherent has no counterpart in spectroscopy. Also, it may be that other kinds of noise, such as detector noise, could be treated within this analogy. Although setup noise is the dominant noise in coherent processing, it is a much lesser factor in achromatic coherent processing, especially when the operation is one-dimensional and the source is a line source, with coherence in only one dimension, and it is overshadowed by other noise in incoherent processing. It appears also to be a minor factor in spectroscopy.

There remains a final significant observation. Noting that the grating vs. interference spectroscopy situation is analogous to the achromatic coherent optical processor vs. incoherent optical processing, and also noting that, in theory, achromatic optical coherent optical processing is better in all respects than the coherent and is at a disadvantage only in the practical sense of being cumbersome and more difficult to implement, it follows that if interference spectroscopy has in some circumstances a competitive advantage over grating spectroscopy, then incoherent optical processing, even for situations where processing operation has a large space bandwidth product ($N \ll 1$), can be advantageous over achromatic optical processing, and even more so over purely coherent processing.

5. A Method for Coherent Fourier Transformation Using Spatially Incoherent Light

This method is still being analyzed, and the complete analysis is rather intricate. Here we described the idea from a rather qualitative viewpoint, using Fig. 5.5, which shows an interferometer consisting of 3 diffraction gratings. The first grating splits the

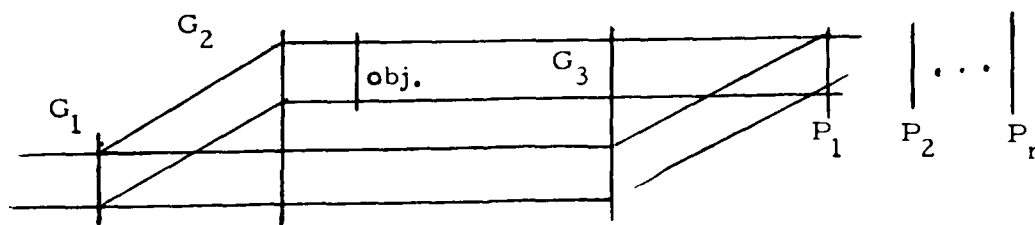


Figure 5.1

incident light into two beams, the second grating renders the two beams parallel, and the third combines the two beams. An object to be recorded as a Fourier transform hologram is shown between G_2 and G_3 . Let the object (only one dimensional, or two dimensional, having one lateral dimension and depth) have amplitude transmittance s , and let its Fourier transform be

$$S(f_x) = \int_{-\infty}^{\infty} s(x) e^{j2\pi f_x x} dx$$

and let it be expressed in the Fourier decomposition form

$$s(z) = \int_{-\infty}^{\infty} S(f_z) e^{-j2\pi f_z z} df_z$$

One component of the Fourier representation, of spatial frequency f_1 , is

$$S(f_1) e^{j2\pi f_1 z}, \text{ or } |S(f_1)| e^{j2\pi f_1 z + \phi(f_1)}$$

where ϕ is the phase of S . We can think of the object as being a superposition of diffraction gratings, each one corresponding to one Fourier component, as in the above equation. With respect to this component, we can think of the interferometer as a 4-grating interferometer, this object component constituting the 4th grating. Under broad source illumination, broad-source fringes will form at a plane to the right of G_3 , where the two beams exactly combine, that is, where rays split off at G_1 are precisely recombined. The fringe pattern has a component with contrast proportional to $|S(f_1)|$, and the phase of the fringes has a term $\phi(f_1)$. In short, both the phase and amplitude of S have been recorded holographically, even though the light is completely incoherent.

Each spatial frequency component of the object similarly produces a fringe pattern, but each in a different plane. For example, the component f_1 forms at a plane P_1 , and f_2 at plane P_2 , as in the above figure. If we now pass a tilted plane through this sequence of planes P_1, P_2, \dots, P_n , this tilted plane will intersect each plane P_n along a single line. We have, by recording in this tilted plane, recorded a Fourier transform of the object s , and in completely incoherent light. Furthermore, if the object is a phase object, we have also recorded the phase of that object. In short, the system behaves just as if the light were coherent. There is, of course, a bias buildup problem, because the light at any plane P_n consists of not only a fringe pattern due to one Fourier component, but also of a bias background due to all other Fourier components. Thus, the fringes are of low contrast, but in principle, the SNR is very high, which at least compensates for the low signal level. It may be, indeed, that the SNR is much

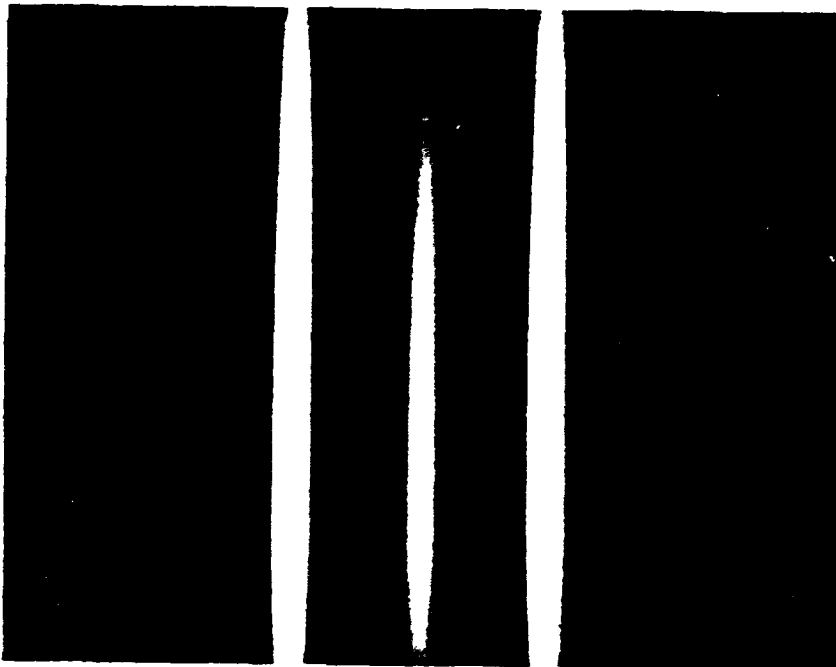
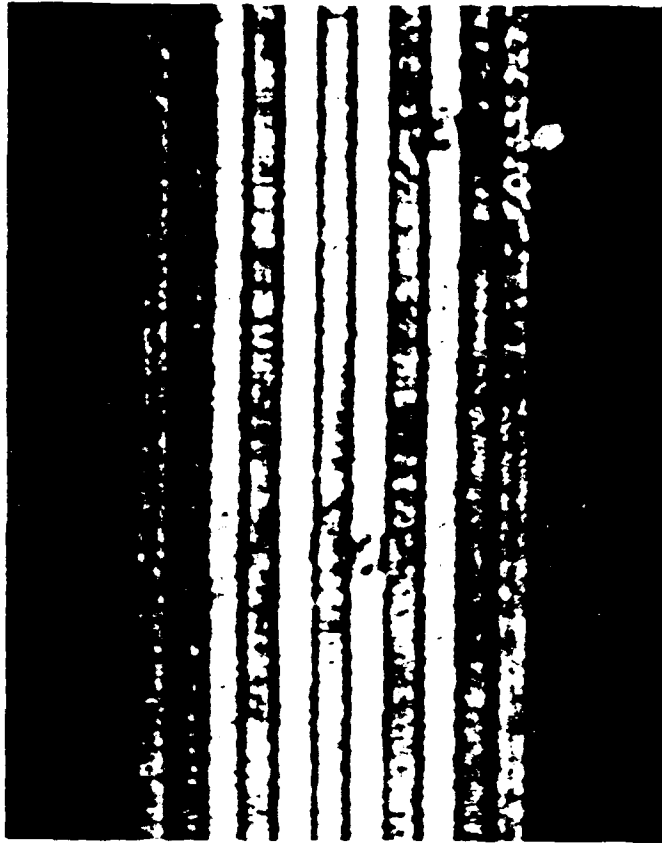
higher than for a coherent system. A mathematical analysis is needed to give quantitative results.

Figure 5.1 shows a reconstruction from such a Fourier transform hologram. The object was a bleached square wave grating, having a transmittance that is partly amplitude and partly phase. Fig. 5.2 shows the image of the Fourier transform hologram, taken in the 1st order. That the 0 order is weaker than the 1st orders shows that the object was either complex or bipolar, i.e., that the Fourier transform is in amplitude rather than intensity.

Figures 5.3 and 5.4 show two reconstructions from a hologram of 3-D object consisting of a split and a wire in different planes. In reconstruction, the wire and the slit focus in different lanes, showing that this method of holography yields a 3-D reconstruction.

This method has come atop a previous idea we were pursuing for doing incoherent spatial filtering using a grating interferometer, and using the signal-bearing mask in front of the source. The Fourier transform is formed as an irradiance distribution, and the transform of the signal mask irradiance rather than amplitude.

Both techniques are being described in papers under preparation, and we defer an extended discussion of these techniques until these papers are completed.



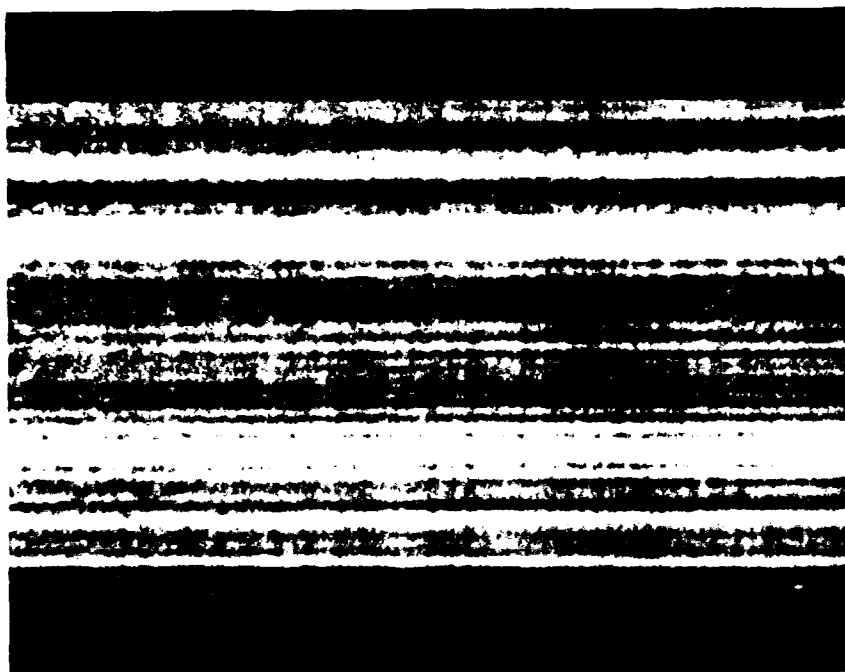


Figure 2

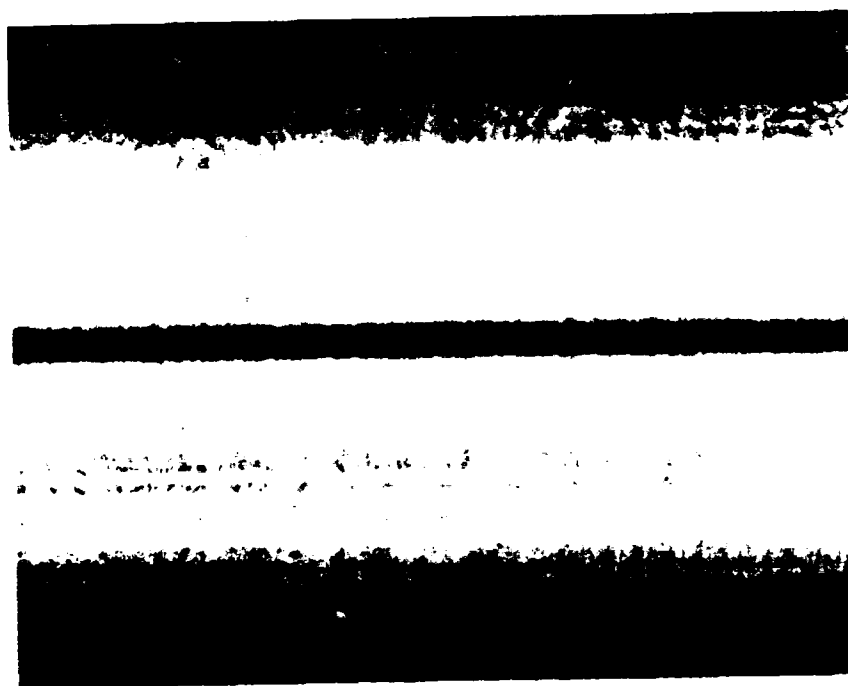


Figure 3

Finally, an idea has been conceived for the imaging of incoherent objects through a phase distorting medium. The source of the distortion is presumed to be localized in some plane between the object and imaging systems, so that spatial invariance of the impulse response may be assumed. The wavefront distortion induced by a turbulent medium can in many instances be characterized in this way. Also, we note that the spatial frequency content of the distortion and its depth of modulation need not be very great to cause significant aberration in the image of a conventional incoherent imaging system. Under these circumstances, the perturbation of the wavefront can be characterized as slowly propagating, i.e., the deviation of the wavefront from the ideal changes slowly with propagation distance, and the wavefront correlates highly with itself over relatively large distances. Thus the primary change in the wavefront over some ranges of distances is predominantly that due to the change in curvature of the ideal wavefront. It follows that if two slightly longitudinally separated wavefronts were brought together and interfered, the wavefront deviation, being initially pure phase and not having propagated far enough to be transferred into amplitude perturbations, would tend to cancel out. The difference in curvature between the two planes would, however, remain.

It can readily be shown that the result of the above-described interference experiment effectively produces a Fresnel transform of the object distribution and thus retains all information about the original object. The Fresnel transform recorded, however, is devoid to a large extent of the effects of the phase distorting medium.

We have developed a hybrid imaging system consisting of conventional imaging and a grating interferometer, to implement the above interference process. The result is a method for imaging through a turbulent atmosphere.

6. A Simple Method for Image Deblurring

Many methods have been described for the restoration of a blurred image by the use of optical processing. Here we describe a method that is both simple and easy to implement, yet gives rather good results.

The blurring process can be described by a point spread function $h(x,y)$, which, for the case of a uniform linear blur, of width L_o , is just $\text{rect}(x/L_o)$. The blurred image, $g(x,y)$, is then related to the unblurred image $s(x,y)$, by

$$g(x,y) = s(x,y) * h(x,y) \quad (1)$$

which Fourier transforms to

$$G(u,v) = S(u,v)H(u,v) \quad (2)$$

where G, S and H are respectively, the Fourier transforms of g, s and h . The deblurring is typically done by spatial filtering, using an inverse filter, $1/H(u,v)$, consisting of an amplitude and a phase part, $H = H \exp(j\theta_H)$. Of the various methods for generating the filter, one first described by Ragnarsson¹ is especially simple and effective, and we have extended this work in several ways

To make the restoration filter for linear blur of width L_o , we place a slit of dimensions L_o and $L_o/8$ in the aperture of a coherent, Fourier transforming optical system, producing

$$H(u,v) = U_o \text{Sinc}(L_o u / \lambda f_o) \text{Sinc}(L_o v / 8 \lambda f_o) \approx U_o \text{Sinc}(L_o u / \lambda f_o) \quad (3)$$

where f_o is the focal length of the transforming lens. Since this pattern is considerably wider in v than in u , we treat it as a one-dimensional pattern, constant in v .

A plane-wave reference beam, $u_r \exp(i\theta_r)$, is introduced, producing the intensity distribution

$$I = U_o^2 \text{Sinc}^2(L_o u / \lambda f_o) + U_r^2 + 2 U_o U_r \text{Sinc}(L_o u / \lambda f_o) \cos \theta_r \quad (4)$$

which describes a fringe pattern with contrast

$$M = \frac{2 U_o U_r \text{Sinc}(L_o u / \lambda f_o)}{U_o^2 \text{Sinc}^2(L_o u / \lambda f_o) + U_r^2} \quad (5)$$

Letting the ratio of reference beam to object beam be ϵ , i.e.

$$\epsilon = U_r / (U_o \text{Sinc}(L_o u / \lambda f_o)) \quad (6)$$

Eq. (5) simplifies to

$$M = 2\epsilon / (1 + \epsilon^2) \quad (7)$$

For $\epsilon \ll 1$, i.e., for a reference beam much weaker than the object beam, Eqs. 5 and 7 become

$$M \approx 2\epsilon = \frac{2U_r}{U_o \text{Sinc}(L_o u / \lambda f_o)} \quad (8)$$

Thus, the spatial filter produced by recording this distribution as a hologram will be approximately the required inverse filter. This result is in sharp contrast to the usual case, $\epsilon \gg 1$, i.e., the reference beam much stronger than the object beam, which leads to the more usual result

$$M \approx 2/\epsilon = 2U_o \text{Sinc}(L_o u / \lambda f_o) / U_r \quad (9)$$

These results assume a linear recording of the irradiance I . Ragnarsson used a monobath developer to expand the linear range up to 250:1 and even better. One shortcoming is that this filter needs to be bleached which generally raises the noise level.

We employed an alternative method, given by Zetzsche, to keep the recording within the linear region of the film². An absorptive mask was placed in front of the recording plate. the mask attenuates the reference and object beam alike. The mask is constructed by recording the object alone, so the mask then attenuates inversely to the strength of the object beam.

The major problem in image deblurring is noise in the deblurred image. There are two kinds of noise:

(a) False images. The inverse filter has poles, which cannot be realized in the actual filter. The failure to record the poles may lead to false images appearing in the output. For a point object, the restored image will typically have a number of weaker false object points lying on either side of the proper image point, at spacing equal to the blur width. These false images are minimized when the inverse filter is closely matched to the blur width. Thus scrupulous matching is important. Our experimental system permitted the scale of the Fourier transform to be

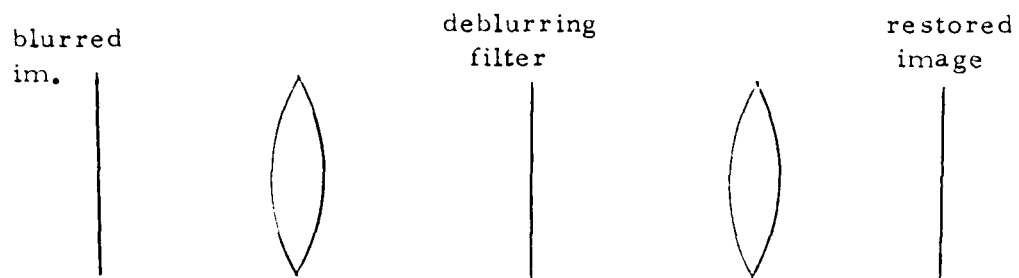


Figure 6.1

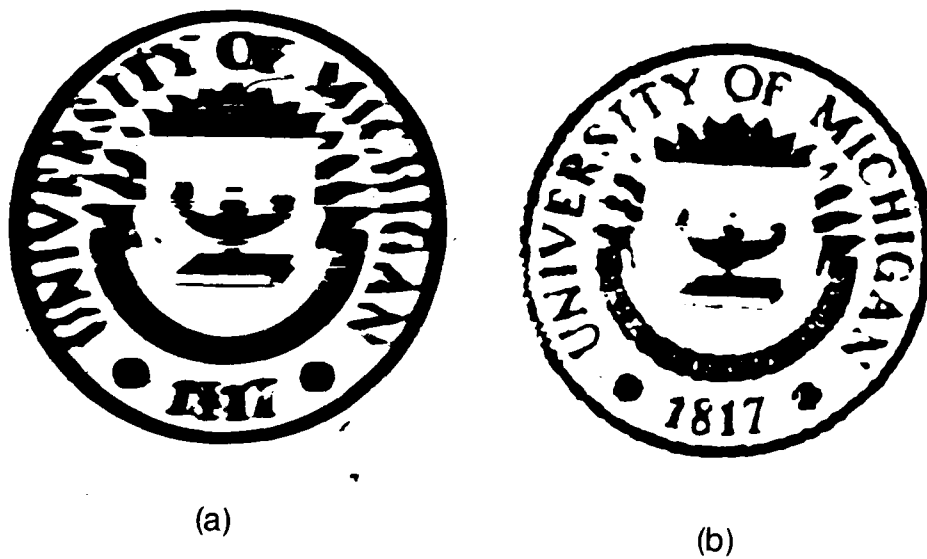


Fig. 6.2. Experimental results. Aa. blurred,
b. deblurred.

adjusted in size, so that it could experimentally be accurately.

(b) System noise. The inverse filter, since it attenuates most where the signal is weakest, inherently accentuates system noise, that is noise such as dust, dirt, and scratches on the lenses, and grain noise of the spatial filter. Thus, extremely clean technique is required; also, techniques such as placing the blurred image transparency in a liquid gate are helpful.

The optical system for making the inverse filter was similar to Zetzsche's, except that provision was made for noise reduction and for scale change. The filter-making process is used twice: First, without the reference beam, the mask $1/|H|$ is produced. Next, a weak reference beam is introduced and a hologram is recorded.

The spatial filtering system is shown in Fig. 6.1. This is a standard system, except that provision is made for varying the scale of the image Fourier transform. This is accomplished by illuminating the object with a wave that can be adjusted to be a plane, diverging, or converging, thereby shifting the plane where the Fourier transform is formed, and also altering the scale of the transform. Thus, the same filter can be used for restoring images with different blur widths.

Finally, since coherence only in the x direction is needed, we destroy the coherence in the y direction; this is done by putting a diffraction grating between source and object. The grating rulings are along the x direction, and the grating is translated continuously in y during the exposure. This greatly reduces the system noise, giving a much improved image. The same noise reduction method is used in the filter making system.

The same filter was used for restoration of images with four different blur widths, in the ratio 4:3:2:1. In the construction, the filter was matched to the blur width 3, but also worked for the other blur widths after appropriate adjustments in the optical system.

Figures 6.2 shows an example of blurred image and its restoration. We note that the SNR of the restored image is rather good.

7. Journal Articles

a. Articles published during this period, but covering work done in the previous (June 1980 to August 1981)

1. Optical Holography with Partially Coherent Radiation, E. Leith. Proceedings of the Conference on Low Energy X-ray Diagnostics, Monterey, Calif., June 1981.
2. Recording of Phase-Amplitude Images, E. Leith and G.J. Swanson, *Optics Letters*, *20* 3081 (Sept. 1981).
3. Achromatic Fourier Transform Holography, G.D. Collins, *Applied Optics*, *20* 3109 (15 Sept 1981).
4. White Light Image Deblurring, G.G. Yang and E.N. Leith, *Applied Optics* *20*, 3995 (Dec. 1, 1982).
5. Recording of One-Dimensionally Dispersed Holograms in White Light, *Applied Optics* *20*, 4267 (15 Dec. 1981).

b. Articles prepared during this reporting period, but not yet published.

1. Holographic Aberration Compensation with Partially Coherent Light, E.N. Leith and G.J. Swanson.. Scheduled for Dec. issue of *Optics Letters*.

2. Bias Levels and Signal to Noise Levels in Incoherent Recording E.N. Leith and G.J. Swanson,. In preparation.
3. A Simple Method for Image Deblurring, by Ya-Guang Jiang and You-Ren Xu, Submitted to Applied Optics
4. Fourier Transformation with Incoherent Light, by G.D. Collins. In preparation

8. Persons Associated with this Effort

E. Leith (PI)

G.J. Swanson (Graduate Student)

G.D. Collins (Graduate Student)

Y.G. Jiang (Visiting Scholar)

Y.S. cheng (Graduate Student)

9. New Discoveries

Of our various accomplishments, two can be classified as our principal discoveries:

1. We discovered a way to do phase conjugation with light of considerably reduced coherence, leading to images of far superior quality in phase conjugation imaging. Experimental results show the method works quite well (See experimental results in sec. 3)
2. We discovered a way to do *coherent* Fourier transformation and holographic recording in spatially incoherent light. Experimental results (shown in sec. 5) show the process works just as theory predicts, and the experimental results are rather good.

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